UNIT 5

Kinematics of Particles

&

Kinetics of particles

UNIT V
KINEMATICS AND KINETICS

A body is said to be in motion if it changes its position with respect to its surroundings. The nature of path of displacement of various particles of a body determines the type of motion. The motion may be of the following types :

1. *Rectilinear translation*
2. *Curvilinear translation*
3. Rotary or circular motion.

Rectilinear translation is also known as straight line motion. Here particles of a body move in straight parallel paths. Rectilinear means forming straight lines and translation means behaviour. Rectilinear translation will mean behaviour by which straight lines are formed. Thus, when a body moves such that its particles form parallel straight paths the body is said to have rectilinear translation.

In a curvilinear translation the particles of a body move along circular arcs or curved paths.

Rotary or circular motion is a special case of curvilinear motion where particles of a body move along concentric circles and the displacement is measured in terms of angle in radians or revolutions.

DEFINITIONS

1. ***Displacement.*** *If a particle has rectilinear motion with respect to some point which is* assumed to be fixed, its displacement is its total change of position during any interval of time. The point of reference usually assumed is one which is at rest with respect to the surfaces of the earth.

The unit of displacement is same as that of distance or length. In M.K.S. or S.I. system it is one metre.

2. Rest and motion. A body is said to be at *rest* at an instant (means a small interval of time) if its position with respect to the surrounding objects remains unchanged during that instant.

A body is said to be in motion at an instant if it changes its position with respect to its surrounding objects during that instant.

Actually, nothing is absolutely at rest or absolutely in motion : all rest or all motion is relative only.

3. ***Speed.*** *The speed of body is defined as its rate of change of its position with respect to its surroundings irrespective of direction. It is a scalar quantity. It is measured by distance covered per unit time.*

Mathematically, speed

$$= \frac{\text{Distance covered}}{\text{Time taken}} = \frac{S}{t}$$

Its units are m/sec or km/hour.

4. Velocity. The velocity of a body is *its rate of change of its position with respect to its surroundings in a particular direction.* It is a *vector quantity*. It is measured by the distance covered in a *particular direction* per unit time.

i.e., $$\text{Velocity} = \frac{\text{Distance covered (in a particular direction)}}{\text{Time taken}}$$

$$v = \frac{S}{t} .$$

Its units are same as that of speed *i.e.*, m/sec or km/hour.

5. Uniform velocity. *If a body travels equal distances in equal intervals of time in the same direction it is said to be moving with a uniform or constant velocity*. If a car moves 50 metres with a constant velocity in 5 seconds, its velocity will be equal to,

$$\frac{50}{5} = 10 \text{ m/s.}$$

6. Variable velocity. *If a body travels unequal distances in equal intervals of time, in the same direction, then it is said to be moving with a variable velocity* or if it is changes either its speed or its direction or both shall again be said to be moving with a variable velocity.

7. Average velocity. *The average or mean velocity of a body is the velocity with which the distance travelled by the body in the same interval of time, is the same as that with the variable velocity*.

If u = initial velocity of the body

v = final velocity of the body

t = time taken

S = distance covered by the body

Then average velocity $= \frac{u + v}{\;}$

and $$S = \left(\frac{u + v}{2}\right) \times t$$

8. Acceleration. The *rate of change of velocity of a body is called its acceleration*. When the velocity is increasing the acceleration is reckoned as *positive*, when decreasing as *negative*. It is represented by a or f.

If u = initial velocity of a body in m/sec

v = final velocity of the body in m/sec

t = time interval in seconds, during which the change has occurred,

Then acceleration, $$a = \frac{v - u}{t} \frac{\text{m/sec}}{\text{sec}}$$

or $$a = \frac{v - u}{t} \text{ m/sec}^2$$

From above, it is obvious that if *velocity of the body remains constant, its acceleration will be zero.*

9. Uniform acceleration. *If the velocity of a body changes by equal amounts in equal intervals of time, the body is said to move with uniform acceleration.*

10. Variable acceleration. *If the velocity of a body changes by unequal amount in equal intervals of time, the body is said to move with variable acceleration.*

DISPLACEMENT-TIME GRAPHS

Refer to Fig (*a*). The graph is parallel to the time-axis indicating that the *displacement is not changing with time*. The *slope of the graph is zero*. The body has no velocity and is at rest.

Refer to Fig. (*b*). The displacement increases linearly with time. The displacement increases by equal amounts in equal intervals of time. *The slope of the graph is constant*. In other words, the body is moving with a *uniform velocity*.

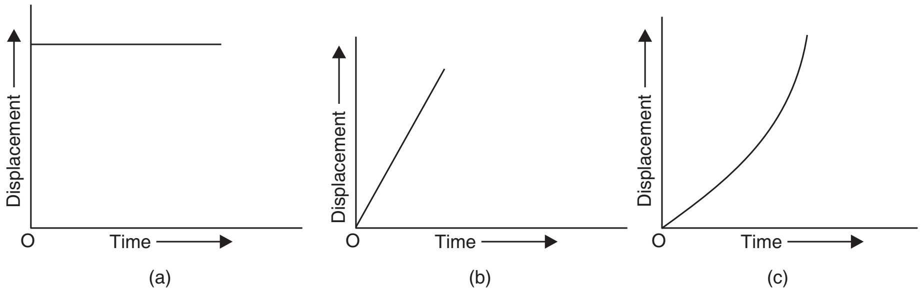


Fig. Displacement-time graphs

Refer to Fig. (*c*). The displacement time graph is a *curve*. This means that the displacement is not changing by equal amounts in equal intervals of time. The slope of the graph is different at different times. In other words, the velocity of the body is changing with time. The motion of the body is accelerated.

7.4. VELOCITY-TIME GRAPHS

Refer to Fig. (*a*). The velocity of the body increases linearly with time. The slope of the graph is constant, *i.e.,* velocity changes by equal amounts in equal intervals of time. In other words, the *acceleration of the body is constant*. Also, at time $t = 0$, the velocity is finite. Thus, the body, *moving with a finite initial velocity, is having a constant acceleration*.

Refer to Fig. (*b*). The body has a finite initial velocity. As the time passes, the velocity decreases linearly with time until its final velocity becomes zero, *i.e.* it comes to rest. Thus, the body has a *constant deceleration* (or retardation) since the *slope of the graph is negative*.

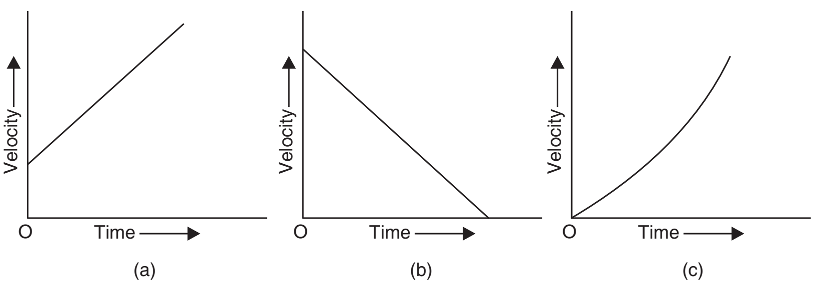


Fig. Velocity-time graphs

Refer to Fig. (*c*). The velocity-time graph is a *curve*. The slope is therefore, different at different times. In other words, the *velocity is not changing at a constant rate. The body does not have a uniform acceleration since the acceleration is changing with time*.

EQUATIONS OF MOTION UNDER UNIFORM ACCELERATION

First Equation of Motion. *Relation between u, v, a and t.*

Let us assume that a body starts with an initial velocity u and acceleration a. After time t, it attains a velocity v. Therefore, the change in velocity in t seconds = $v - u$. Hence, the change in velocity in one second = $\frac{v-u}{t}$. By definition, this is equal to the acceleration a.

Thus, $$a = \frac{v-u}{t}$$

or $$at = v - u$$

or $$v = u + at$$

Second Equation of Motion. *Relation between S, u, a and t.*

Let a body moving with an initial uniform velocity u is accelerated with a uniform acceleration a for time t. After time t its final velocity is v. The distance S which the body travels in time t is determined as follows :

Now, since the acceleration is uniform, *i.e.*, the velocity changes by an equal amount in equal intervals of time, it is obvious that the average velocity is just the average of initial and final velocities.

$$\text{Average velocity} = \left(\frac{u+v}{2}\right)$$

∴ Distance travelled = average velocity × time

$$S = \left(\frac{u+v}{2}\right) \times t$$

or $$S = \left(\frac{u+u+at}{2}\right) \times t \qquad (\because \quad v = u + at)$$

$$= \left(u + \frac{at}{2}\right) \times t$$

or $$S = ut + \frac{1}{2}at^2$$

Third Equation of Motion. *Relation u, v, a and S.* We know, that

$$S = \text{average velocity} \times \text{time}$$

$$= \left(\frac{u+v}{2}\right) \times t$$

$$= \left(\frac{u+v}{2}\right) \times \left(\frac{v-u}{a}\right) \qquad \left(\because \quad t = \frac{v-u}{a}\right)$$

$$= \frac{v^2 - u^2}{2a}$$

∴ $$v^2 - u^2 = 2aS$$

DISTANCE COVERED IN nth SECOND BY A BODY MOVING WITH UNIFORM ACCELERATION

Let u = initial velocity of the body

a = acceleration

S_{nth} = distance covered in nth second

then $S_{nth} = \begin{pmatrix}\text{distance covered} \\ \text{in } n \text{ second, } s_n\end{pmatrix} - \begin{pmatrix}\text{distance covered in } (n-1) \\ \text{second, } s_{n-1}\end{pmatrix}$

Using the relation,

$$S_n = un + \frac{1}{2}an^2 \qquad (\because \quad t = n)$$

and

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$= u(n-1) + \frac{1}{2}a(n^2 - 2n + 1)$$

$\therefore$

$$S_{nth} = S_n - S_{n-1}$$

$$= \left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n^2 - 2n + 1)\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - a/2$$

$$= u + an - a/2$$

$\therefore$

$$S_{nth} = u + a/2(2n - 1)$$

1. *A car accelerates from a velocity of 36 km/hour to a velocity of 108 km/hour in a distance of 240 m. Calculate the average acceleration and time required.*

Sol. Initial velocity,

$$u = 36 \text{ km/hour}$$

$$= \frac{36 \times 1000}{60 \times 60} = 10 \text{ m/sec}$$

Final velocity, $v = 108$ km/hour

$$= \frac{108 \times 1000}{60 \times 60} = 30 \text{ m/sec}$$

Distance, $S = 240$ m.

Average acceleration, a = ?

Using the relation,

$$v^2 - u^2 = 2aS$$

$$(30)^2 - (10)^2 = 2 \times a \times 240$$

or

$$900 - 100 = 480\,a$$

or

$$a = \frac{800}{480} = \mathbf{1.67\ m/sec^2.\ (Ans.)}$$

Time required, $\boldsymbol{t}$ **= ?**

$$v = u + at$$

$$30 = 10 + 1.67 \times t$$

$\therefore$

$$t = \frac{(30 - 10)}{1.67} = \mathbf{11.97\ sec.\ (Ans.)}$$

2. *A body has an initial velocity of 16 m/sec and an acceleration of 6 m/sec^2. Determine its speed after it has moved 120 metres distance. Also calculate the distance the body moves during 10th second.*

Sol. Initial velocity, $u = 16$ m/sec

Acceleration, $a = 6$ m/sec^2

Distance, $S = 120$ metres

Speed, $\boldsymbol{v = ?}$

Using the relation,

$$v^2 - u^2 = 2aS$$

$$v^2 - (16)^2 = 2 \times 6 \times 120$$

or

$$v^2 = (16)^2 + 2 \times 6 \times 120$$

$$= 256 + 1440 = 1696$$

$$\boldsymbol{v = 41.18 \text{ m/sec.}} \textbf{ (Ans.)}$$

Distance travelled in 10th sec ; S_{10th} = ?

Using the relation,

$$S_{nth} = u + \frac{a}{2}(2n - 1)$$

$$S_{10th} = 16 + \frac{6}{2}(2 \times 10 - 1) = 16 + 3(20 - 1)$$

$$= \textbf{73 m.} \quad \textbf{(Ans.)}$$

3. *On turning a corner, a motorist rushing at 15 m/sec, finds a child on the road 40 m ahead. He instantly stops the engine and applies brakes, so as to stop the car within 5 m of the child, calculate : (i) retardation, and (ii) time required to stop the car.*

Sol. Initial velocity, $u = 15$ m/sec

Final velocity, $v = 0$

Distance, $S = 40 - 5 = 35$ m.

(*i*) **Retardation,** $\boldsymbol{a = ?}$

Using the relation,

$$v^2 - u^2 = 2aS$$

$$0^2 - 15^2 = 2 \times a \times 35$$

$$\therefore \quad \boldsymbol{a = -3.21 \text{ m/sec}^2.} \quad \textbf{(Ans.)}$$

[– ve sign indicates that the acceleration is negative, *i.e.,* retardation]

(*ii*) **Time required to stop the car, t = ?**

Using the relation,

$$v = u + at$$

$$0 = 15 - 3.21 \times t \qquad (\because \; a = -3.21 \text{ m/sec}^2)$$

$$\therefore \quad \boldsymbol{t} = \frac{15}{3.21} = \textbf{4.67 s.} \quad \textbf{(Ans.)}$$

4. *A burglar's car had a start with an acceleration 2 m/sec^2. A police vigilant party came after 5 seconds and continued to chase the burglar's car with a uniform velocity of 20 m/sec. Find the time taken, in which the police will overtake the car.*

Sol. Let the police party overtake the burglar's car in t seconds, after the instant of reaching the spot.

Distance travelled by the burglar's car in t seconds, S_1 :

Initial velocity, $u = 0$

Acceleration, $a = 2$ m/sec^2

Time, $t = (5 + t)$ sec.

Using the relation,

$$S = ut + \frac{1}{2}at^2$$

$$S_1 = 0 + \frac{1}{2} \times 2 \times (5 + t)^2$$

$$= (5 + t)^2 \quad ...(i)$$

Distance travelled by the police party, S_2 :

Uniform velocity, $v = 20$ m/sec.

Let t = time taken to overtake the burglar's car

$\therefore$ Distance travelled by the party,

$$S_2 = v \times t = 20t \quad ...(ii)$$

For the police party to overtake the burglar's car, the two distances S_1 and S_2 should be equal.

i.e.,

$$S_1 = S_2$$

$$(5 + t)^2 = 20t$$

$$25 + t^2 + 10t = 20t$$

$$t^2 - 10t + 25 = 0$$

$\therefore$
$$t = \frac{+10 \pm \sqrt{100 - 100}}{2}$$

or **t = 5 sec. (Ans.)**

5. *A car starts from rest and accelerates uniformly to a speed of 80 km/hour over a distance of 500 metres. Calculate the acceleration and time taken.*

If a further acceleration raises the speed to 96 km/hour in 10 seconds, find the acceleration and further distance moved.

The brakes are now applied and the car comes to rest under uniform retardation in 5 seconds. Find the distance travelled during braking.

Sol. Considering the **first period of motion :**

Initial velocity, $u = 0$

Velocity attained, $v = \frac{80 \times 1000}{60 \times 60} = 22.22$ m/sec.

Distance covered, $S = 500$ m

If a is the acceleration and t is the time taken,

Using the relation :

$$v^2 - u^2 = 2aS$$

$$(22.22)^2 - 0^2 = 2 \times a \times 500$$

$\therefore \quad a = \dfrac{(22.22)^2}{2 \times 500} = \mathbf{0.494\ m/sec^2.}$ **(Ans.)**

Also, $\quad v = u + at$

$22.22 = 0 + 0.494 \times t$

$\therefore \quad \boldsymbol{t} = \dfrac{22.22}{0.494} = \mathbf{45\ sec.}$ **(Ans.)**

Now considering the **second period of motion,**

Using the relation,

$$v = u + at$$

where $\quad v = 96 \text{ km/hour} = \dfrac{96 \times 1000}{60 \times 60} = 26.66 \text{ m/sec}$

$u = 80 \text{ km/hour} = 22.22 \text{ m/sec}$

$t = 10 \text{ sec}$

$\therefore \quad 26.66 = 22.22 + a \times 10$

$\therefore \quad \boldsymbol{a} = \dfrac{26.66 - 22.22}{10} = \mathbf{0.444\ m/sec^2.}$ **(Ans.)**

To calculate distance covered, using the relation

$$S = ut + \frac{1}{2} at^2$$

$$= 22.22 \times 10 + \frac{1}{2} \times 0.444 \times 10^2$$

$$= 222.2 + 22.2 = 244.4$$

$\therefore \quad \boldsymbol{S} = \mathbf{244.4\ m.}$ **(Ans.)**

During the period when brakes are applied :

Initial velocity, $\quad u = 96 \text{ km/hour} = 26.66 \text{ m/sec}$

Final velocity, $\quad v = 0$

Time taken, $\quad t = 5 \text{ sec.}$

Using the relation,

$$v = u + at$$

$$0 = 26.66 + a \times 5$$

$\therefore \quad a = \dfrac{-26.66}{5} = -5.33 \text{ m/sec}^2.$

(–ve sign indicates that acceleration is negative *i.e.*, retardation)

Now using the relation,

$$v^2 - u^2 = 2aS$$

$$0^2 - (26.66)^2 = 2 \times -5.33 \times S$$

$\therefore \quad S = \dfrac{26.66^2}{2 \times 5.33} = 66.67 \text{ m.}$

$\therefore$ **Distance travelled during braking = 66.67 m. (Ans.)**

6. *Two trains A and B moving in opposite directions pass one another. Their lengths are 100 m and 75 m respectively. At the instant when they begin to pass, A is moving at 8.5 m/sec with a constant acceleration of 0.1 m / sec² and B has a uniform speed of 6.5 m / sec. Find the time the trains take to pass.*

Sol. Length of train $A = 100$ m

Length of train $B = 75$ m

∴ Total distance to be covered

$= 100 + 75 = 175$ m

Imposing on the two trains *A* and *B*, a velocity equal and opposite to that of *B*.

Velocity of train $A = (8.5 + 6.5) = 15.0$ m/sec

and velocity of train $B = 6.5 - 6.5 = 0.$

Hence the train *A* has to cover the distance of 175 m with an acceleration of 0.1 m/sec^2 and an initial velocity of 15.0 m/sec.

Using the relation,

$$S = ut + \frac{1}{2}at^2$$

$$175 = 15t + \frac{1}{2} \times 0.1 \times t^2$$

$$3500 = 300t + t^2$$

or $$t^2 + 300t - 3500 = 0$$

$$t = \frac{-300 \pm \sqrt{90000 + 14000}}{2} = \frac{-300 \pm 322.49}{2}$$

$$= 11.24 \text{ sec.}$$

Hence the trains take 11.24 seconds to pass one another. (Ans.)

7. *The distance between two stations is 2.6 km. A locomotive starting from one station, gives the train an acceleration (reaching a speed of 40 km/h in 0.5 minutes) until the speed reaches 48 km/hour. This speed is maintained until brakes are applied and train is brought to rest at the second station under a negative acceleration of 0.9 m/sec². Find the time taken to perform this journey.*

Sol. Considering the motion of the locomotive *starting from the first station*.

Initial velocity $u = 0$

Final velocity $v = 40$ km/hour

$$= \frac{40 \times 1000}{60 \times 60} = 11.11 \text{ m/sec.}$$

Time taken, $t = 0.5$ min or 30 sec.

Let 'a' be the resulting acceleration.

Using the relation,

$$v = u + at$$

$$11.11 = 0 + 30a$$

∴ $$a = \frac{11.11}{30} = 0.37 \text{ m/sec}^2.$$

Let t_1 = time taken to attain the speed of 48 km/hour

$$\left(\frac{48 \times 1000}{60 \times 60} = 13.33 \text{ m/sec.}\right)$$

Again, using the relation,

$$v = u + at$$

$$13.33 = 0 + 0.37t_1$$

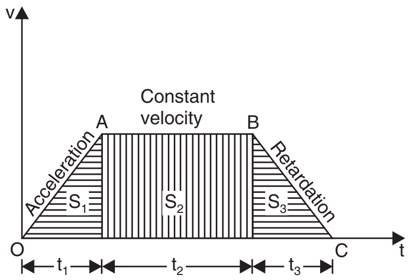
$$\therefore \quad t_1 = \frac{13.33}{0.37} = 36 \text{ sec.} \qquad ...(i)$$

and the distance covered in this interval is given by the relation,

$$S_1 = ut_1 + \frac{1}{2}at_1^2$$

$$= 0 + \frac{1}{2} \times 0.37 \times 36^2 = \mathbf{240\ m.}$$

Now, considering the motion of the *retarding period* before the locomotive comes to rest at the second station (*i.e.*, portion *BC* in Fig. 7.3).



Now, $u = 13.33$ m/sec

$v = 0$

$a = -0.9$ m/sec^2

Let $t = t_3$ be the time taken

Using the relation,

$$v = u + at$$

$$0 = 13.33 - 0.9t_3$$

$$\therefore \quad t_3 = \frac{13.33}{0.9} = 14.81 \text{ sec} \qquad ...(ii)$$

and distance covered,

$$S_3 = \text{average velocity} \times \text{time}$$

$$= \left(\frac{13.33 + 0}{2}\right) \times 14.81 = 98.7 \text{ m}$$

∴ Distance covered with constant velocity of 13.33 m/sec,

$$S_2 = \text{total distance between two stations} - (S_1 + S_2)$$

$$= (2.6 \times 1000) - (240 + 98.7) = 2261.3 \text{ m.}$$

∴ Time taken to cover this distance,

$$t_2 = \frac{2261.3}{13.33} = 169.6 \text{ sec} \qquad ...(iii)$$

Adding (*i*), (*ii*) and (*iii*)

Total time taken,

$$T = t_1 + t_2 + t_3$$

$$= 36 + 169.6 + 14.81$$

$$\mathbf{= 220.41\ sec.} \quad \textbf{(Ans.)}$$

8. *Two trains A and B leave the same station on parallel lines. A starts with a uniform acceleration of 0.15 m/sec^2 and attains a speed of 24 km / hour when the steam is required to keep speed constant. B leaves 40 seconds after with uniform acceleration of 0.30 m/sec^2 to attain a maximum speed of 48 km/hour. When will B overtake A ?*

Sol. Motion of train *A* :

Uniform acceleration, $a_1 = 0.15$ m/sec^2

Initial velocity, $u_1 = 0$

Final velocity, $v_1 = 24$ km/hour

$$= \frac{24 \times 1000}{60 \times 60} = \frac{20}{3} \text{ m/sec.}$$

Let t_1 be the time taken to attain this velocity (in seconds).

Using the relation,

$$v = u + at$$

$$\frac{20}{3} = 0 + 0.15t_1$$

$$\therefore \quad t_1 = \frac{20}{3 \times 0.15} = 44.4 \text{ sec.}$$

Also, distance travelled during this interval,

$$S_1 = ut_1 + \frac{1}{2}at_1^2$$

$$= 0 + \frac{1}{2} \times 0.15 \times 44.4^2$$

$$= 148 \text{ m.}$$

Motion of train B :

Initial velocity, $u_2 = 0$

Acceleration, $a_2 = 0.3$ m/sec^2

Final velocity, $v_2 = 48$ km/hr

$$= \frac{48 \times 1000}{60 \times 60} = \frac{40}{3} \text{ m/sec.}$$

Let t_2 be the time taken to travel this distance, say S_2.

Using the relation,

$$v = u + at$$

$$\frac{40}{3} = 0 + 0.3t_2$$

$$\therefore \quad t_2 = \frac{40}{3 \times 0.3} = 44.4 \text{ sec}$$

and

$$S_2 = u_2t_2 + \frac{1}{2}a_2 t_2^2$$

$$= 0 + \frac{1}{2} \times 0.3 \times (44.4)^2$$

$$= 296 \text{ m.}$$

Let the train *B* overtake the train *A* when they have covered a distance *S* from the start. And let the train *B* take *t* seconds to cover the distance.

Thus, time taken by the train $A = (t + 40)$ sec.

Total distance moved by train A,

$$S = 148 + \text{distance covered with constant speed}$$

$$S = 148 + [(t + 40) - t_1]\ \frac{20}{3}$$

$$= 148 + [t + 40 - 44.4] \times \frac{20}{3}$$

$$= 148 + (t - 4.4) \times \frac{20}{3} \qquad ...(i)$$

[{$(t + 40) - t_1$} is the time during which train A moves with constant speed]

Similarly, total distance travelled by the train B,

$$S = 296 + \text{distance covered with constant speed}$$

$$= 296 + (t - 44.4) \times \frac{40}{3} \qquad ...(ii)$$

Equating (i) and (ii),

$$148 + (t - 4.4)\ \frac{20}{3} = 296 + (t - 44.4) \times \frac{40}{3}$$

$$148 + \frac{20}{3}\ t - \frac{88}{3} = 296 + \frac{40}{3} t - \frac{1776}{3}$$

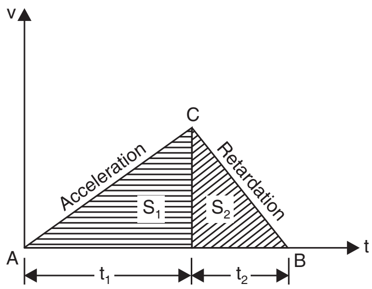
$$\left(\frac{40}{3} - \frac{20}{2}\right) t = 148 - 296 + \frac{1776}{3} - \frac{88}{3}$$

or $$t = 62.26 \text{ sec.}$$

Hence, the train B overtakes the train A after 62.26 sec. of its start. (Ans.)

Two stations A and B are 10 km apart in a straight track, and a train starts from A and comes to rest at B. For three quarters of the distance, the train is uniformly accelerated and for the remainder uniformly retarded. If it takes 15 minutes over the whole journey, find its acceleration, its retardation and the maximum speed it attains.

Sol. Refer to Fig. 7.4.



Distance between A and B,

$$S = 10 \text{ km} = 10{,}000 \text{ m}$$

Considering the motion in the first part :

Let u_1 = initial velocity = 0

a_1 = acceleration

t_1 = time taken

S_1 = distance travelled.

Using the relation,

$$S = ut + \frac{1}{2}\ at^2$$

$$S_1 = 0 + \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_1 t_1^2 \qquad ...(i)$$

$$7500 = \frac{1}{2} a_1 t_1^2 \qquad ...(ii)$$

[$\because$ S_1 = 3/4 × 10,000 = 7500 m]

Also, for the second retarding part

$$u_2 = \text{initial velocity}$$
$$= \text{final velocity at the end of first interval}$$
$$= 0 + a_1t_1 = a_1t_1$$

Hence v_2 = final velocity at the end of second part

$$= u_2 - a_2t_2$$
$$= a_1t_1 - a_2t_2$$
$$= 0, \text{ because the train comes to rest}$$

$\therefore$ $a_1t_1 = a_2t_2$

or $$\frac{a_1}{a_2} = \frac{t_2}{t_1} \qquad ...(iii)$$

Also, S_2 = distance travelled in the second part

$$= \text{average velocity} \times \text{time}$$
$$= \left(\frac{a_1t_1 + 0}{2}\right) \times t_2$$
$$= \frac{a_1t_1}{2} \cdot t_2 \qquad ...(iv)$$

Adding (*i*) and (*iv*),

$$S_1 + S_2 = \frac{a_1t_1^2}{2} + \frac{a_1t_1}{2} \cdot t_2$$
$$= \frac{a_1t_1}{2}(t_1 + t_2)$$

or $$S_1 + S_2 = \frac{a_1t_1}{2} \times 900 \qquad (\because\ t_1 + t_2 = 15 \text{ min.} = 900 \text{ sec})$$

$$10{,}000 = \frac{a_1t_1}{2} \times 900 \qquad (\because\ S_1 + S_2 = 10 \text{ km} = 10{,}000 \text{ m})$$

or $$a_1t_1 = \frac{20{,}000}{900} = \frac{200}{9}$$

But a_1t_1 = maximum velocity

Hence **max. velocity** $= \frac{200}{9}$ **= 22.22 m/sec (Ans.)**

Also, from eqn. (*ii*)

$$7500 = \frac{1}{2} \times 22.22 \times t_1$$

or $$t_1 = \frac{7500}{11.11} = 675 \text{ sec}$$

$\therefore$ $t_2 = 900 - 675 = 225$ sec

Now from eqn. (*iii*),

$$\frac{a_1}{a_2} = \frac{t_2}{t_1} = \frac{225}{675} = \frac{1}{3}$$

$\therefore$ $3a_1 = a_2$.

Also, $v_{max} = 22.22 = a_1 t_1$

$\therefore$ $$\boldsymbol{a_1} = \frac{22.22}{675} = \mathbf{0.0329\ m/sec^2}. \quad \textbf{(Ans.)}$$

and $\boldsymbol{a_2} = 3a_1$

$= 3 \times 0.0329$

$= \mathbf{0.0987\ m/sec^2}$. **(Ans.)**

MOTION UNDER GRAVITY

It has been seen that bodies falling to earth (through distances which are small as compared to the radius of the earth) and entirely unrestricted, increase in their velocity by about 9.81 m/sec for every second during their fall. This acceleration is called the acceleration due to gravity and is conventionally denoted by '*g*'. Though the value of this acceleration varies a little at different parts of the earth's surface but the generally adopted value is 9.81 m/sec^2.

For downward motion	**For upward motion**
$a = +g$	$a = -g$
$v = u + gt$	$v = u - gt$
$h = ut + \frac{1}{2}gt^2$	$h = ut - \frac{1}{2}gt^2$
$v^2 - u^2 = 2gh$	$v^2 - u^2 = -2gh.$

SOME HINTS ON THE USE OF EQUATIONS OF MOTION

(*i*) If a body starts from rest, its initial velocity, $u = 0$

(*ii*) If a body comes to rest ; its final velocity, $v = 0$

(*iii*) When a body is thrown upwards with a velocity u, time taken to reach the maximum height $= \frac{u}{g}$ and velocity on reaching the maximum height is zero (*i.e.*, $v = 0$) . This value of t is obtained by equating $v = u - gt$ equal to zero.

(*iv*) Greatest height attained by a body projected upwards with a velocity $u = \frac{u^2}{2g}$, which is obtained by substituting $v = 0$ in the equation $v^2 - u^2 = -2gh$.

(*v*) Total time taken to reach the ground $= \frac{2u}{g}$, the velocity on reaching the ground being $\sqrt{2gh}$.

($\because$ $v^2 - u^2 = 2gh$ or $v^2 - 0^2 = 2gh$ or $v = \sqrt{2gh}$)

(*vi*) *The velocity with which a body reaches the ground is same with which it is thrown upwards.*

10. *A stone is dropped from the top of tower 100 m high. Another stone is projected upward at the same time from the foot of the tower, and meets the first stone at a height of 40 m. Find the velocity, with which the second stone is projected upwards.*

Sol. Motion of the first particle :

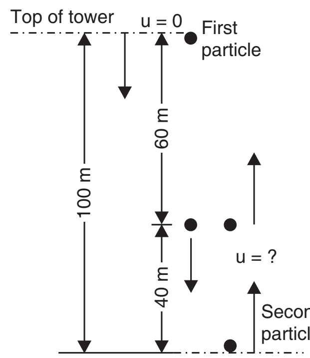
Height of tower $= 100$ m

Initial velocity, $u = 0$

Height, $h = 100 - 40 = 60$ m.

Let t be the time (in seconds) when the two particles meet after the first stone is dropped from the top of the tower.

Refer to Fig. 7.5.



Using the relation,

$$h = ut + \frac{1}{2} gt^2$$

$$60 = 0 + \frac{1}{2} \times 9.81\ t^2$$

$$\therefore \qquad t = \sqrt{\frac{120}{9.81}} = 3.5 \text{ sec.}$$

Motion of the second particle :

Height, $h = 40$ m

Time, $t = 3.5$ sec.

Let u be the initial velocity with which the second particle has been projected upwards.

Using the relation,

$$h = ut - \frac{1}{2} gt^2 \qquad (\because \text{ Particle is projected upwards})$$

$$40 = u \times 3.5 - \frac{1}{2} \times 9.81 \times 3.5^2$$

$$3.5u = 40 + \frac{1}{2} \times 9.81 \times 3.5^2$$

$$\boldsymbol{u = 28.6 \text{ m/sec.}} \quad \textbf{(Ans.)}$$

11. *A body projected vertically upwards attains a maximum height of 450 m. Calculate the velocity of projection and compute the time of flight in air. At what altitude will this body meet a second body projected 5 seconds later with a speed of 140 m/sec ?*

Sol. Maximum height attained by the body

= 450 m

Let u = initial velocity of the body

v = final velocity of the body = 0

Using the relation,

$$v^2 - u^2 = -2gh \qquad (\because \text{ body is thrown upwards})$$

$$0^2 - u^2 = -2 \times 9.81 \times 450$$

$$\boldsymbol{u = 94 \text{ m/sec.}} \ \textbf{(Ans.)}$$

Let 't' be the time taken by the body in reaching the highest point from the point of projection.

Then, using the relation,

$$v = u - gt$$

$$0 = 94 - 9.81t$$

$$\therefore \qquad t = \frac{94}{9.81} = 9.6 \text{ sec.}$$

∴ Total time of flight in air

= 2 × 9.6 = **19.2 sec. (Ans.)**

(∵ The body will take the same time in returning also)

Let the second body meet the first body at a height 'h' from the ground. Let 't' be the time taken by the first body.

Then, time taken by the second body

$$= (t - 4) \text{ sec.}$$

Considering the motion of first body

$$h = ut - \frac{1}{2} gt^2$$

$$= 94t - \frac{1}{2} \times 9.81t^2 \quad ...(i)$$

Considering the motion of the second body

$$h = 140 (t - 5) - \frac{1}{2} \times 9.81 (t - 5)^2 \quad ...(ii)$$

Equating (*i*) and (*ii*), we get

$$94t - \frac{1}{2} \times 9.81t^2 = 140 (t - 5) - \frac{1}{2} \times 9.81 (t - 5)^2$$

$$188t - 9.81t^2 = 280 (t - 5) - 9.81 (t - 5)^2$$

$$188t - 9.81t^2 = 280t - 1400 - 9.81 (t - 5)^2$$

$$188t - 9.81t^2 = 280t - 1400 - 9.81t^2 + 98.1t - 245.25$$

From which $t = 8.65$ sec.

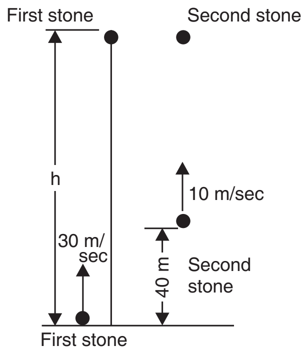
Putting this in eqn. (*i*), we get

$$h = 94 \times 8.65 - \frac{1}{2} \times 9.81 \times 8.65^2$$

$$= 813.3 - 367 = 446.3 \text{ m.}$$

Hence, the second body will meet the first one at a height of 446.3 m from the ground. (Ans.)

12. *Two stones are thrown vertically upwards one from the ground with a velocity of 30 m/sec and another from a point 40 metres above with a velocity of 10 m/sec. When and where will they meet ?*



Sol. Refer to Fig.

Let the two stones meet after 't' seconds from their start at a height of 5 metres from the ground.

Motion of first stone :

u = initial velocity = 30 m/sec

h = vertical distance travelled

t = time taken

Using the relation, $h = ut - \frac{1}{2} gt^2$

($\because$ stone is thrown upwards)

$$h = 30t - \frac{1}{2} \times 9.81t^2 \quad ...(i)$$

Motion of second stone :

Vertical distance travelled

$$h' = h - 40$$
$$u = 10 \text{ m/sec.}$$

Again using the relation,

$$h = ut + \frac{1}{2} gt^2$$

$$(h - 40) = 10t - \frac{1}{2} \times 9.8t^2 \qquad ...(ii)$$

Subtracting (*ii*) from (*i*),

$$40 = 20t$$

$$\boldsymbol{t = 2 \text{ sec.}} \quad \textbf{(Ans.)}$$

Substituting this value in eqn. (*i*), we get

$$h = 30 \times 2 - \frac{1}{2} \times 9.81 \times 2^2 = \mathbf{40.38\ m.\ (Ans.)}$$

Hence, the two stones meet after 2 seconds at 40.38 m from the ground.

13. *A stone is thrown from the ground vertically upwards, with a velocity of 40 m/sec. After 3 seconds another stone is thrown in the same direction and from the same place. If both of the stones strike the ground at the same time, compute the velocity with which the second stone was thrown.*

Sol. Motion of first stone :

u = velocity of projection = 40 m/sec

v = velocity at the maximum height = 0

t = time taken to reach the maximum height = ?

Using the relation,

$$v = u - gt \qquad (\because \text{ stone is moving upward})$$
$$0 = 40 - 9.81t$$

or $$t = \frac{40}{9.81} = 4 \text{ sec.}$$

Therefore, total time taken by the first stone to return to the earth = 4 + 4 = 8 sec (because the time taken to reach the maximum height is same as that to come down to earth).

Therefore, the time taken by the second stone to return to the earth = 8 – 3 = 5 sec.

or time taken to reach the maximum height = 5/2 = 2.5 sec.

Motion of second stone :

u = velocity of projection = ?

v = final velocity at max. height = 0

t = time taken to reach the max. height

Using the relation,

$$v = u - gt$$
$$0 = u - 9.81 \times 2.5$$

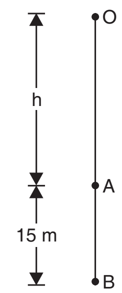
$\therefore$ $$u = 9.81 \times 2.5 = 24.5 \text{ m/sec.}$$

Hence, the velocity of projection of second stone

= 24.5 m/sec. (Ans.)

14. *A body, falling freely under the action of gravity passes two points 15 metres apart vertically in 0.3 seconds. From what height, above the higher point, did it start to fall.*

Sol. Refer to Fig. 7.7.



Let the body start from O and pass two points A and B, 15 metres apart in 0.3 second after traversing the distance OA.

Let $OA = h$

Considering the motion from O to A,

Initial velocity, $u = 0$

Using the relation,

$$h = ut + \frac{1}{2} gt^2 \quad (\because \text{ the body is falling downward})$$

$$h = 0 + \frac{1}{2} gt^2 \quad ...(i)$$

Considering the motion from O to B.

Initial velocity, $u = 0$

Time taken, $t = (t + 0.3)$ sec.

Again, using the relation, $h + 15 = 0 + \frac{1}{2} g\,(t + 0.3)^2 \quad ...(ii)$

Subtracting, (i) from (ii),

$$15 = \frac{1}{2} g(t + 0.3)^2 - \frac{1}{2} gt^2$$

$$30 = g(t^2 + 0.6t + 0.09) - gt^2$$

$$30 = gt^2 + 0.6\,gt + 0.09\,g - gt^2$$

$$\therefore \quad 0.6gt = 30 - 0.09g$$

$$t = \frac{30}{0.6g} - \frac{0.09g}{0.6g} = 5.1 - 0.15 = 4.95 \text{ sec.} \quad ...(iii)$$

Substituting the value of t in eqn. (i), we get

$$\boldsymbol{h} = \frac{1}{2} \times 9.81 \times (4.95)^2 = \mathbf{120.2\ m.} \quad \textbf{(Ans.)}$$

15. *A stone dropped into a well is heard to strike the water after 4 seconds. Find the depth of the well, if the velocity of sound is 350 m/sec.*

Sol. Initial velocity of stone, $u = 0$

Let t = time taken by stone to reach the bottom of the well,

and h = depth of the well

Using the relation,

$$h = ut + \frac{1}{2} gt^2$$

$$h = 0 + \frac{1}{2} \times 9.8t^2 = 4.9t^2 \quad ...(i)$$

Also, the time taken by the sound to reach the top

$$= \frac{\text{Depth of the well}}{\text{Velocity of sound}}$$

$$= \frac{h}{350} = \frac{4.9t^2}{350} \quad ...(ii)$$

Total time taken = time taken by the stone to reach the bottom of the well

+ time taken by sound to reach the ground

= 4 seconds (given)

$$\therefore \quad t + \frac{4.9t^2}{350} = 4$$

or $$4.9t^2 + 350t - 1400 = 0$$

or $$t = \frac{-350 \pm \sqrt{(350)^2 + 4 \times 4.9 \times 1400}}{2 \times 4.9}$$

$$= \frac{-350 + 387.2}{9.8} = 3.8 \text{ sec}$$

$$\therefore \quad t = 3.8 \text{ sec.}$$

Substituting the value in eqn. (*i*), we get

$$h = 4.9 \times (3.8)^2 = 70.8 \text{ m}$$

Hence, the depth of well = 70.8 m. (Ans.)

VARIABLE ACCELERATION

16. *The equation of motion of a particle is* $S = -6 - 5t$

$^2 + t^3$

where S is in metres and t in seconds.

Calculate : (*i*) *The displacement and the acceleration when the velocity is zero.*

(*ii*) *The displacement and the velocity when the acceleration is zero.*

Sol. The equation of motion is

$$S = -6 - 5t^2 + t^3 \quad ...\text{(given)} \quad ...(i)$$

Differentiating both sides,

$$\frac{ds}{dt} \quad \text{or} \quad v = -10t + 3t^2$$

$$\therefore \quad v = -10t + 3t^2 \quad ...(ii)$$

Again, differentiating both sides,

$$\frac{dv}{dt} \quad \text{or} \quad a = -10 + 6t$$

$$\therefore \quad a = -10 + 6t \quad ...(iii)$$

Now, **(*i*) When the velocity is zero,**

$$v = -10t + 3t^2 = 0$$

$$\therefore \quad t(3t - 10) = 0$$

$$t = \frac{10}{3} = 3.33 \text{ sec.} \quad \text{(ignoring } t = 0 \text{ which means start)}$$

Substituting this value in eqns. (*i*) and (*iii*),

$$S = \text{displacement}$$
$$= -6 - 5 \times 3.33^2 + 3.33^3$$
$$= -6 - 55.44 + 36.92$$
$$= \mathbf{-24.52\ m.} \quad \textbf{(Ans.)}$$

The negative sign indicates that distance is travelled in the other direction.

Also, a = acceleration

$$= -10 + 6 \times \frac{10}{3} = \mathbf{10\ m/sec^2.} \quad \textbf{(Ans.)}$$

(*ii*) When the acceleration is zero

$$a = -10 + 6t = 0$$

$\therefore$ $6t = 10$

or $t = \frac{10}{6} = \frac{5}{3} = 1.67$ sec.

Substituting this value in eqns. (*i*) and (*ii*), we get

$$S = \text{displacement}$$
$$= -6 - 5t^2 + t^3 = -6 - 5 \times (1.67)^2 + (1.67)^3$$
$$= -6 - 13.94 + 4.66 = \mathbf{-15.28\ m.} \quad \textbf{(Ans.)}$$

The –ve sign again means that the distance is travelled in the other direction.

Also, $v = -10t + 3t^2$

$$= -10 \times 1.67 + 3 \times (1.67)^2 = -16.7 + 8.36$$
$$= \mathbf{-8.34\ m/sec.} \quad \textbf{(Ans.)}$$

17. *If a body be moving in a straight line and its distance S in metres from a given point in the line after t seconds is given by the equation*

$$S = 20t + 3t^2 - 2t^3.$$

Calculate : (a) The velocity and acceleration at the start.

(b) The time when the particle reaches its maximum velocity.

(c) The maximum velocity of the body.

Sol. The equation of motion is

$$S = 20t + 3t^2 - 2t^3 \qquad ...(i)$$

Differentiating both sides

$$\frac{dS}{dt} = v = 20 + 6t - 6t^2 \qquad ...(ii)$$

Again, differentiating

$$\frac{d^2S}{dt^2} = \frac{dv}{dt} = a = 6 - 12t \qquad ...(iii)$$

(*a*) At start, *t* = 0

Hence from eqns. (*ii*) and (*iii*),

$$\boldsymbol{v} = 20 + 0 - 0 = \mathbf{20\ m/sec.} \quad \textbf{(Ans.)}$$
$$\boldsymbol{a} = 6 - 12 \times 0 = \mathbf{6\ m/sec.} \quad \textbf{(Ans.)}$$

(*b*) When the particle reaches its maximum velocity

$$a = 0$$

i.e., $$6 - 12t = 0$$

$\therefore$ $$\boldsymbol{t = 0.5 \text{ sec.}} \quad \textbf{(Ans.)}$$

(*b*) The maximum velocity of the body

When $t = 0.5$ sec.

$$v_{max} = 20 + 6t - t^2$$
$$= 20 + 6 \times 0.5 - 6 \times 0.5^2$$
$$= 20 + 3 - 1.5$$
$$= \textbf{21.5 m/sec.} \quad \textbf{(Ans.)}$$

SELECTED QUESTIONS EXAMINATION PAPERS

18. *Two trains A and B leave the same station on parallel lines. A starts with uniform acceleration of 0.15 m/s^2 and attains a speed of 24 km/hour when the steam is reduced to keep the speed constant. B leaves 40 seconds after with a uniform acceleration of 0.30 m/s^2 to attain a maximum speed of 48 km/hour. When will B overtake A ?*

Sol. Motion of train A:

Uniform acceleration, $a_1 = 0.15$ m/s^2

Initial velocity, $u_1 = 0$

Final velocity, $v_1 = 24$ km/h

$$= \frac{24 \times 1000}{60 \times 60} = \frac{20}{3} \text{ m/sec}$$

Let t_1 be the time taken to attain this velocity (in seconds)

Using the relation:

$$v = u + at$$

$$\frac{20}{3} = 0 + 0.15 \times t_1$$

$\therefore$ $$t_1 = \frac{20}{3 \times 0.15} = 44.4 \text{ sec}$$

Also, distance travelled during this interval,

$$s_1 = u_1 t_1 + \frac{1}{2} a_1 t_1^2$$

$$= 0 + \frac{1}{2} \times 0.15 \times 44.4^2 = 148 \text{ m}$$

Motion of train B:

Initial velocity, $u_2 = 0$

Acceleration, $a_2 = 0.3$ m/sec^2

Final velocity, $v_2 = 48$ km/h

$$= \frac{48 \times 1000}{60 \times 60} = \frac{40}{3} \text{ m/sec}$$

Let t_2 be taken to travel this distance, say s_2

Using the relation:

$$v = u + at$$

$$\frac{40}{3} = 0 + 0.3 \times t_2$$

$\therefore$ $$t_2 = \frac{40}{3 \times 0.3} = 44.4 \text{ s}$$

and $$s_2 = u_2 t_2 + \frac{1}{2} a_2 t_2^2$$

$$= 0 + \frac{1}{2} \times 0.3 \times (44.4)^2 = 296 \text{ m}$$

Let the train *B* overtake the train *A* when they have covered a distance *s* from the start. And let the train *B* take *t* seconds to cover the distance.

Thus, time taken by the train $A = (t + 40)$ sec.

Total distance moved by train *A*.

$$s = 148 + \text{distance covered with constant speed}$$
$$= 148 + [(t + 40) - t_1] \times 20/3$$
$$= 148 + [t + 40 - 44.4] \times 20/3$$
$$= 148 + (t - 4.4) \times 20/3 \quad ...(i)$$

[$\{(t + 40) - t_2\}$ is the time during which train *A* moves with constant speed].

Similarly, total distance travelled by the train *B*,

$$s = 296 + \text{distance covered with constant speed}$$
$$= 296 + (t - 44.4) \times 40/3 \quad ...(ii)$$

Equating (*i*) and (*ii*)

$$148 + (t - 4.4) \times 20/3 = 296 + (t - 44.4) \times 40/3$$

$$148 + \frac{20}{3}t - \frac{88}{3} = 296 + \frac{40}{3}t - \frac{1776}{3}$$

$$\left(\frac{40}{3} - \frac{20}{3}\right)t = 148 - 296 + \frac{1776}{3} - \frac{88}{3}$$

$$\boldsymbol{t = 62.26 \text{ s}}$$

Hence, *train B, overtakes train A after 62.26 s of its start.* **(Ans.)**

19. *A cage descends a mine shaft with an acceleration of 1 m/s^2. After the cage has travelled 30 m, stone is dropped from the top of the shaft. Determine: (i) the time taken by the stone to hit the cage, and (ii) distance travelled by the cage before impact.*

Sol. Acceleration of cage,

$$a = 1 \text{ m/s}^2$$

Distance travelled by the shaft before dropping of the stone = 30 m

(*i*) Time taken by the stone to hit the cage = ?

Considering motion of the stone.

Initial velocity, $u = 0$

Let t = time taken by the stone to hit the cage, and

h_1 = vertical distance travelled by the stone before the impact.

Using the relation,

$$h = ut + \frac{1}{2} gt^2$$

$$h_1 = 0 + \frac{1}{2} \times 9.8\, t^2 = 4.9\, t^2 \quad ...(i)$$

Now let us consider motion of the cage for 30 m

Initial velocity, $u = 0$

Acceleration, $a = 1.0\ m/s^2$.

Let t' = time taken by the shaft to travel 30 m

Using the relation,

$$s = ut + \frac{1}{2} at^2$$

$$30 = 0 + \frac{1}{2} \times 1 \times (t')^2$$

$$t' = 7.75\ s.$$

It means that cage has travelled for 7.75 s before the stone was dropped. Therefore total time taken by the cage before impact = $(7.75 + t)$.

Again using the relation:

$$s = ut + \frac{1}{2} at^2$$

$$s_1 = 0 + \frac{1}{2} \times 1 \times (7.75 + t)^2 \qquad ...(ii)$$

In order that stone may hit the cage the two distances must be equal *i.e.,* equating (*i*) and (*ii*).

$$4.9\ t^2 = \frac{1}{2} \times (7.75 + t)^2$$

$$4.9 = 0.5\ (60 + t^2 + 15.5\ t)$$

or $$9.8 = t^2 + 15.5\ t + 60$$

or $$t^2 + 15.5\ t - 50.2 = 0$$

$$t = \frac{-15.5 \pm \sqrt{(15.5)^2 + 4 \times 50.2}}{2} = \frac{-15.5 \pm \sqrt{441.05}}{2}$$

$$= \frac{-15.5 \pm 21.0}{2} = 2.75\ s \qquad \text{(neglecting –ve sign)}$$

$\therefore$ **t = 2.75 s. (Ans.)**

(*ii*) Distance travelled by the cage before impact = ?

Let s_2 = distance travelled by the cage before impact.

We know total time taken by the cage before impact.

$$= 7.75 + 2.75 = 10.5\ s.$$

Now using the relation,

$$s_2 = ut + \frac{1}{2} at^2$$

$$= 0 + \frac{1}{2} \times 1 \times (10.5)^2 = 55.12\ m$$

Hence distance travelled by the cage before impact **= 55.12 m. (Ans.)**

8.9. D' ALEMBERT'S PRINCIPLE

D' Alembert, a French mathematician, was the first to point out that on the lines of *equation of static equilibrium, equation of dynamic equilibrium can also be established by introducing inertia force in the direction opposite the acceleration in addition to the real forces on the plane.*

Static equilibrium equations are :

$$\Sigma H \text{ (or } P_x) = 0, \Sigma V \text{ (or } \Sigma P_y) = 0, \Sigma M = 0$$

Similarly when different external forces act on a system in motion, the algebraic sum of all the forces (including the *inertia force*) is zero. This is explained as under :

We know that, $P = ma$ (Newton's second law of motion)

or $P - ma = 0$ or $P + (- ma) = 0$

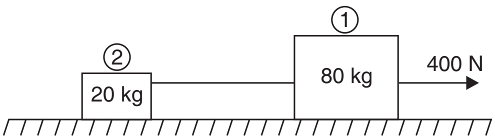
The expression in the block ($- ma$) is the *inertia force* and negative sign signifies that it acts in a direction opposite to that of acceleration/retardation a.

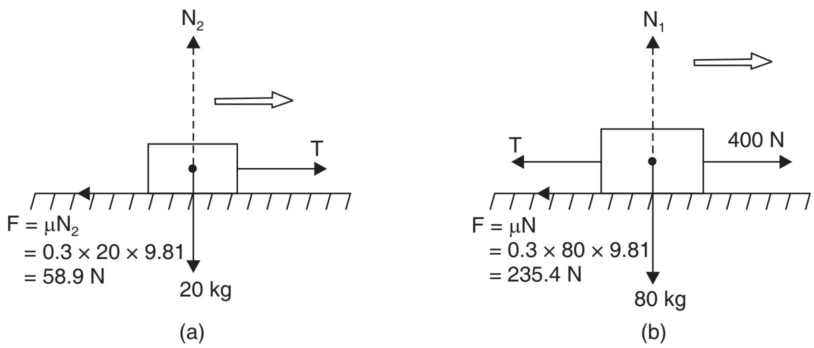
It is also known as the "*principle of kinostatics*".

Example 8.15. *Two bodies of masses 80 kg and 20 kg are connected by a thread and move along a rough horizontal surface under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig. 8.6. The co-efficient of friction between the sliding surfaces of the bodies and the plane is 0.3.*

Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.

Sol. Refer to Figs. 8.5 and 8.6





Acceleration of the bodies, a :

As per *D' Alembert's principle for dynamic equilibrium condition the algebraic sum of all the active forces acting on a system should be zero.*

The various forces acting on the bodies are :

(i) Force applied $= 400$ N

(ii) Inertia force $= (80 + 20)\, a$

(iii) Frictional force $= 0.3 \times 80 \times 9.81 + 0.3 \times 20 \times 9.81$

$= 235.4 + 58.9 = 294.3$ N

$\therefore \quad 400 - (80 - 20)\, a = 294.3 = 0$

or
$$a = \frac{400 - 294.3}{(80 + 20)} = \mathbf{1.057\ m/s^2.\ (Ans.)}$$

Tension in the thread between the two masses, *T* :

Considering free body diagrams of the masses 80 kg and 20 kg separately as shown in Fig. (a) and (b).

Applying D' Alembert's principle for Fig. 8.6 (a), we get

$$400 - T - 80 \times 1.057 - 0.3 \times 80 \times 9.81 = 0$$

$\therefore$ **T = 80 N. (Ans.)**

Now, applying D' Alembert's principle for Fig. 8.6 (b), we get

$$T - 0.3 \times 20 \times 9.81 - 20 \times 1.057 = 0$$

$\therefore$ **T = 80 N. (Ans.)**

It may be noted that the same answer is obtained by considering the two masses separately.

MOTION OF A LIFT

Consider a lift (elevator or cage etc.) carrying some mass and moving with a uniform acceleration.

Let m = mass carried by the lift in kg,

W (= $m.g$) = weight carried by the lift in newtons,

a = uniform acceleration of the lift, and

T = tension in the cable supporting the lift.

There could be the following *two* cases :

(i) When the lift is moving *upwards*, and

(ii) When the lift is moving *downwards*.

1. Lift moving upwards :

Refer to Fig. 8.7.

The net upward force, which is responsible for the motion of the lift

$= T - W = T - m.g$ ...(i)

Also, this force = mass × acceleration

$= m.a$ ...(ii)

Equating (i) and (ii), we get

$T - m.g = m.a$

$\therefore \quad T = m.a + m.g = m(a + g)$ ...(8.4)

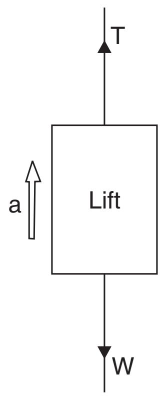


Fig. 8.7. Lift moving upwards.

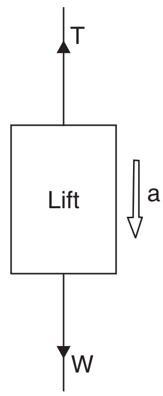


Fig. 8.8. Lift moving downwards.

2. Lift moving downwards :

Refer to Fig. 8.8.

Net downward force responsible for the motion of the lift

$= W - T = m.g - T$ ...(i)

Also, this force = mass × acceleration

$= m.a$ ...(*ii*)

Equating (*i*) and (*ii*), we get

$$m.g - T = m.a$$

$$\therefore \quad T = m.g - m.a = m\,(g - a)$$

16. *An elevator cage of mass 900 kg when empty is lifted or lowered vertically by means of a wire rope. A man of mass 72.5 kg is standing in it. Find :*

(*a*) *The tension in the rope,*

(*b*) *The reaction of the cage on the man, and*

(*c*) *The force exerted by the man on the cage, for the following two conditions :*

(*i*) *when the cage is moving up with an acceleration of 3 m/s*^{2} *and*

(*ii*) *when the cage is moving down with a uniform velocity of 3 m/s.*

Sol. Mass of the cage, $M = 900$ kg

Mass of the man, $m = 72.5$ kg.

(*i*) **Upward acceleration, $a = 3$ m/s^2**

(*a*) Let T be the tension in the rope in newtons

The various forces acting on the cage are :

1. Tension, T of the rope acting vertically upwards.
2. Total mass $= M + m$, of the cage and the man acting vertically downwards.

As the cage moves upwards, $T > (M + m)\,g$

$\therefore$ Net accelerating force $= T - (M + m)\,g = (m + m)\,a$

$$\therefore \quad T - (M + m)\,g = (M + m)\,a \quad ...(i)$$

Substituting the given values, we get

$$T - (900 + 72.5)\,9.81 = (900 + 72.5) \times 3$$

$\therefore$ **T = 12458 N. (Ans.)**

(*b*) Let 'R' be the reaction of the cage on the man in newtons.

Considering the various forces, the equation of motion is

$$R - mg = m.a \quad ...(ii)$$

or $R = mg + ma = m\,(g + a)$

$= 72.5\,(9.81 + 3) =$ **928.7 N. (Ans.)**

(*c*) The force exerted by the man on the cage must be equal to the force exerted by the cage on the man (Newton's third law of motion).

$\therefore$ Force exerted by the man on the cage = **928.7 N. (Ans.)**

(*ii*) **When the cage moves with a uniform velocity 3 m/s :**

When the cages moves with a uniform velocity, acceleration is equal to zero.

(*a*) Tension in the rope, T :

Putting $a = 0$ in eqn. (*i*), we get

$$T - (M + m)\,g = (M + m) \times 0 = 0$$

$$\therefore \quad T = (M + m)\,g$$

$= (900 + 72.5) \times 9.81 =$ **9540 N. (Ans.)**

(*b*) Also from equation (*ii*)

When $a = 0$,

$$R = mg + m \times 0 = mg$$
$$= 72.5 \times 9.81 = \mathbf{711.2\ N.\ (Ans.)}$$

(*c*) Force exerted by the man on the cage

= force exerted by the cage on the man

= **711.2 N. (Ans.)**

17. *An elevator of mass 500 kg is ascending with an acceleration of 3 m/s*^{2}. *During this ascent its operator whose mass is 70 kg is standing on the scales placed on the floor. What is the scale reading ? What will be total tension in the cables of the elevator during his motion ?*

Sol. Mass of the elevator, $M = 500$ kg

Acceleration, $a = 3$ m/s^2

Mass of the operator, $m = 70$ kg

Pressure (R) exerted by the man, when the lift moves upward with an acceleration of 3 m/s^2,

$$R = mg + ma = m\,(g + a)$$
$$= 70\,(9.81 + 3) = \mathbf{896.7\ N.\ (Ans.)}$$

Now, tension in the cable of elevator

$$T = M\,(g + a) + m\,(g + a)$$
$$= (M + m)\,(g + a)$$
$$= (500 + 70)\,(9.81 + 3) = \mathbf{7301.7\ N.\ (Ans.)}$$

MOTION OF TWO BODIES CONNECTED BY A STRING PASSING OVER A SMOOTH PULLEY

Fig. 8.9 shows two bodies of weights W_1 and W_2 respectively hanging vertically from a *weightless* and *inextensible* string, passing over a *smooth pulley.* Let T be the common tension in the string. If the *pulley were not smooth, the tension would have been different in the two sides of the string.*

Let W_1 be greater than W_2 and a be the acceleration of the bodies and their motion as shown.

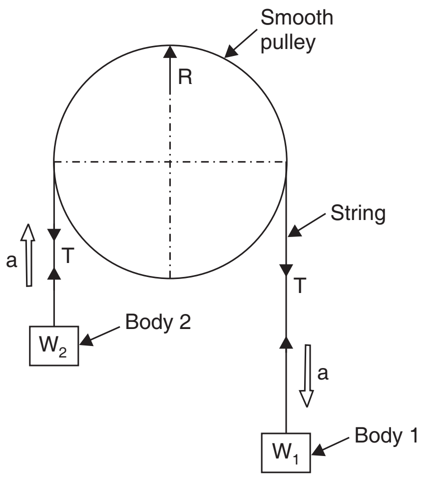


Fig. 8.9

Consider the motion of body 1:

Forces acting on it are : W_1 (downwards) and T (upwards).

$\therefore$ Resulting force = $W_1 - T$ (downwards) ...(*i*)

Since this weight is moving downward, therefore, force acting on this weight

$$= \frac{W_1}{g} \cdot a \qquad ...(ii)$$

Equating (*i*) and (*ii*)

$$W_1 - T = \frac{W_1}{g}\,a \qquad ...(1)$$

Now consider the motion of body 2:

Forces acting on it are : T (upwards) W_2 (downwards)

$\therefore$ Resultant force = $T - W_2$ ...(*iii*)

Since the body is moving upwards therefore force acting on the body

$$= \frac{W_2}{g} \cdot a \qquad ...(iv)$$

Equating (*iii*) and (*iv*)

$$T - W_2 = \frac{W_2}{g} \cdot a \qquad ...(2)$$

Now adding eqns. (1) and (2), we get

$$W_1 - W_2 = \left(\frac{W_1 + W_2}{g}\right) a$$

from which,

$$a = \left(\frac{W_1 + W_2}{W_1 + W_2}\right) . g$$

From equation (2),

$$T - W_2 = \frac{W_2}{g} a$$

$$T = W_2 + \frac{W_2}{g} a = W_2 \left(1 + \frac{a}{g}\right)$$

Substituting the value of 'a' from equation (8.6), we get

$$T = W_2 \left[1 + \left(\frac{W_1 - W_2}{W_1 + W_2}\right) . \frac{g}{g}\right]$$

from which,

$$T = \frac{2\,W_1\,W_2}{W_1 + W_2}$$

Reaction of the pulley,

$$R = T + T = 2T$$

$$= \frac{4W_1\,W_2}{W_1 + W_2}$$

Example 8.18. *Two bodies weighing 45 N and 60 N are hung to the ends of a rope, passing over a frictionless pulley. With what acceleration the heavier weight comes down ? What is the tension in the string ?*

Sol. Weight of heavier body, W_1 = 60 N

Weight of lighter body, W_2 = 45 N

Acceleration of the system, a = ?

Using the relation,

$$a = \frac{g\,(W_1 - W_2)}{(W_1 + W_2)} = \frac{9.81(60 - 45)}{(60 + 45)} = \textbf{1.4 m/s}^2\textbf{. (Ans.)}$$

Tension in the string, T = ?

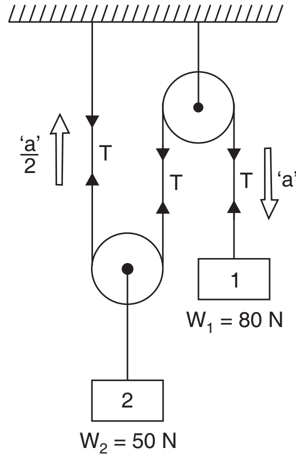
Using the relation,

$$T = \frac{2\,W_1\,W_2}{W_1 + W_2} = \frac{2 \times 60 \times 45}{(60 + 45)} = \textbf{51.42 N. (Ans.)}$$

Example 8.19. *A system of frictionless pulleys carries two weights hung by inextensible cords as shown in Fig. . Find :*

(*i*) *The acceleration of the weights and tension in the cords.*

(*ii*) *The velocity and displacement of weight '1' after 5 seconds from start if the system is released from rest.*



Sol. Weight, $W_1 = 80$ N

Weight, $W_2 = 50$ N

Let T = tension (constant throughout the cord, because pulleys are *frictionless*, and *cord is continuous*).

When weight W_1 travels unit distance then weight W_2 travels half the distance. Acceleration is proportional to the distance.

$\therefore$ If a = acceleration of weight W_1

then, $a/2$ = acceleration of weight W_2.

It is clear from the figure that weight W_1 moves downward and weight W_2 moves upward.

(*i*) **Acceleration of weights, T = ?**

Consider the motion of weight W_1 :

$$W_1 - T = \frac{W_1}{g} a$$

$$80 - T = \frac{80}{g} \times a \qquad ...(i)$$

Consider the motion of weight W_2 :

$$2T - W_2 = \frac{W_2}{g}$$

$$2T - 50 = \frac{50}{g} \times \frac{a}{2} \qquad ...(ii)$$

Multiplying eqn. (*i*) by 2 and adding eqns. (*i*) and (*ii*), we get

$$110 = \frac{185}{g} a$$

$$\therefore \quad a = \frac{110 \times 9.81}{185} = 5.8 \text{ m/s}^2$$

Hence **acceleration of W_1 = 5.8 m/s². (Ans.)**

and **acceleration of W_2** = 5.8/2 = **2.9 m/s². (Ans.)**

Substituting the value of 'a' in eqn. (*i*), we get

$$80 - T = \frac{80}{9.81} \times 5.8$$

$$\therefore \quad \boldsymbol{T = 32.7 \text{ N. (Ans.)}}$$

(*ii*) **Velocity and displacement of weight W_1 after 5 sec. = ?**

$$u = 0,\ a = 5.8 \text{ m/s}^2,\ t = 5 \text{ s}$$

$$\therefore \quad v = u + at = 0 + 5.8 \times 5 = \mathbf{29 \text{ m/s. (Ans.)}}$$

and $$s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} \times 5.8 \times 5^2 = \mathbf{72.5 \text{ m. (Ans.)}}$$

MOTION OF TWO BODIES CONNECTED AT THE EDGE OF A HORIZONTAL SURFACE

Fig. 8.11 shows two bodies of weights W_1 and W_2 respectively connected by a light inextensible string. Let the body 1 hang free and body 2 be placed on a rough horizontal surface. Let the body 1 move downwards and the body 2 move along the surface of the plane. We know that the velocity and acceleration of the body will be the same as that of the body 2, therefore tension will be same throughout the string. Let μ be the co-efficient of friction between body 2 and the horizontal surface.

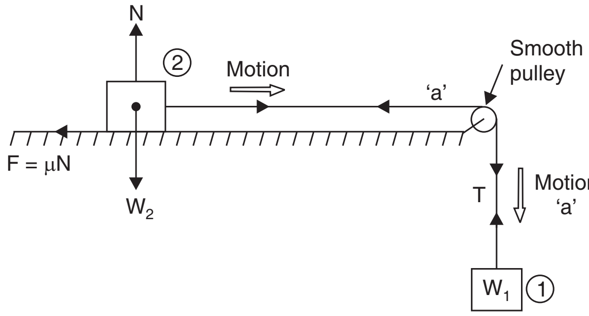


Fig. 8.11

Normal reaction at the surface, $N = W_2$

and force of friction, $F = \mu N = \mu W_2$

Let a = acceleration of the system

T = tension in the string.

Consider the motion of body 1 :

Forces acting on it are : W_1 (downwards) and T (upwards)

Resultant force $= W_1 - T$ ...(*i*)

Since the body is moving downwards, therefore force acting on this body

$$= \frac{W_1}{g} \cdot a \quad \text{...(ii)}$$

Equating (*i*) and (*ii*), $W_1 - T = \frac{W_1}{g} a$ ...(1)

Now consider the motion of body 2 :

Forces acting on it are : T (towards right), Force of friction F (towards left).

$\therefore$ Resultant force $= T - F = T - \mu W_2$ ...(*iii*)

Since, the body is moving horizontally with acceleration, therefore force acting on this body

$$= \frac{W_2}{g} \cdot a \quad \text{...(iv)}$$

Equating (*iii*) and (*iv*), we get

$$T - \mu W_2 = \frac{W_2}{g} a \quad \text{...(2)}$$

Adding equations (1) and (2), we get

$$W_1 - \mu W_2 = \frac{W_1}{g} a + \frac{W_2}{g} a$$

or $$W_1 - \mu W_2 = \frac{a}{g} (W_1 + W_2)$$

or $$a = \left(\frac{W_1 - \mu W_2}{W_1 + W_2}\right) g$$

Substituting this value of 'a' in equation (1), we get

$$W_1 - T = \frac{W_1}{g}\left(\frac{W_1 - \mu W_2}{W_1 + W_2}\right) g$$

$$T = W_1 - W_1\left(\frac{W_1 - \mu W_2}{W_1 + W_2}\right)$$

$$T = W_1 \left[1 - \frac{W_1 - \mu W_2}{W_1 + W_2}\right]$$

$$= W_1 \left[\frac{W_1 + W_2 - W_1 + \mu W_2}{W_1 + W_2}\right]$$

i.e., $$T = \frac{W_1 W_2 (1 + \mu)}{W_1 + W_2}$$

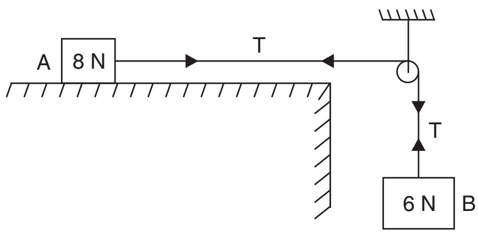
For *smooth horizontal surface* ; putting $\mu = 0$ in equations (8.9) and (8.10), we get

$$a = \frac{W_1 \cdot g}{W_1 + W_2}$$

and $$T = \frac{W_1 W_2}{W_1 + W_2}$$

20. *Find the acceleration of a solid body A of weight 8 N, when it is being pulled by another body of weight 6 N along a smooth horizontal plane as shown in Fig. 8.12.*

Sol. Refer to Fig.



Weight of body B, $W_1 = 6$ N

Weight of body A, $W_2 = 8$ N

Acceleration of body, a = ?

Tension in the string, T = ?

Equation of motion for body B

$$6 - T = \frac{6}{g} \cdot a \quad ...(i)$$

Equation of motion for body A

$$T = \frac{8}{g} \cdot a \quad ...(ii)$$

235 Adding (i) and (ii), we get

$$6 = \frac{14}{g} \cdot a$$

$$\therefore \quad \boldsymbol{a} = \frac{6 \times 9.81}{14} = \mathbf{4.2\ m/s^2}. \textbf{ (Ans.)}$$

Substituting this value of a in (i), we get

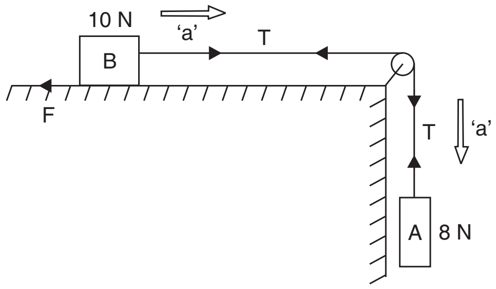
$$6 - T = \frac{6}{9.81} \times 4.2$$

$$\therefore \quad \boldsymbol{T} = \mathbf{3.43\ N}. \textbf{ (Ans.)}$$

21. *Two blocks shown in Fig. have weights A = 8 N and B = 10 N and co-efficient of friction between the block A and horizontal plane,* $\mu = 0.2$.

If the system is released, from rest and the block A falls through a vertical distance of 1.5 m, what is the velocity acquired by it ? Neglect the friction in the pulley and extension of the string.

Sol. Refer to Fig. 8.13.



Considering vertical string portion:

$$8 - T = \frac{8}{g} \cdot a \qquad \text{...}(i)$$

Considering horizontal string portion :

$$T - F = \frac{10}{g} \cdot a$$

or $$T - \mu N_B = \frac{10}{g} \cdot a$$

or $$T - 0.2 \times 10 = \frac{10}{g} a \qquad (\because \; N_B = W_B = 10 \text{ newtons})$$

or $$T - 2 = \frac{10}{g} a \qquad \text{...}(ii)$$

Adding (i) and (ii)

$$6 = \frac{18a}{g}$$

$$\therefore \quad a = \frac{6 \times 9.81}{18} = 3.27 \text{ m/s}^2$$

Now using the relation :

$$v^2 - u^2 = 2as \quad \text{or} \quad v^2 - u^2 = 2 \times 3.27 \times 1.5$$

$$\therefore \quad v = 3.13 \text{ m/s}$$

Hence the velocity acquired by weight $\boldsymbol{A} = \mathbf{3.13\ m/s}$. **(Ans.)**

22. *A body '1' of weight 20 N is held on a rough horizontal table. An elastic string connected to the body '1' passes over a smooth pulley at the end of the table and then under a second smooth pulley carrying a body '2' of weight 10 N as shown in Fig. 8.14. The other end of the string is fixed to a point above the second pulley. When the 20 N body is released, it moves with an accelera-tion of g/5. Determine the value of co-efficient of friction between the block and the table.*

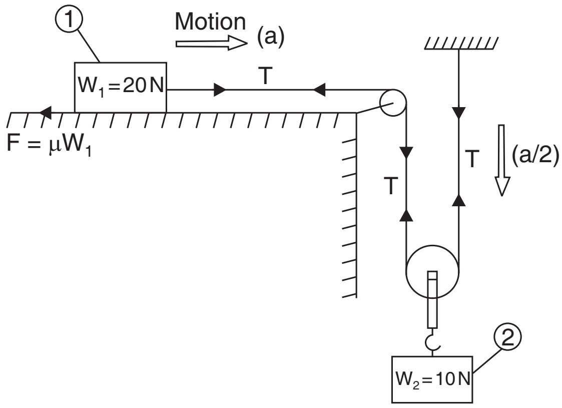
Sol. Weight of body '1', W_1 = 20 N

Weight of body '2', W_2 = 10 N

Acceleration of body '1' $a = g/5$

Let T = tension in string in newtons, and

μ = co-efficient of friction between block and the table.



Considering the motion of body '1' :

$$T - \mu W_1 = \frac{W_1}{g} a$$

or $$T - \mu \times 20 = \frac{20}{g} \times \frac{g}{5} = 4 \quad ...(i)$$

Considering the motion of body '2' :

A little consideration will show that the acceleration of the body '2' will be half of that of the body '1' *i.e.*, $g/10$.

Now, $$W_2 - 2T = \frac{W_2}{g} \times \frac{a}{2}$$

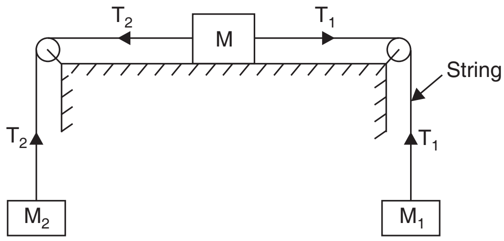
or $$10 - 2T = \frac{10}{g} \times \frac{g}{10} = 1 \quad ...(ii)$$

Now multiplying eqn. (*i*) by 2 and adding eqns. (*i*) and (*ii*), we get

$$10 - 40\mu = 9$$

$\therefore$ $40\mu = 1$ or μ = **0.025. (Ans.)**

Example 8.23. *A string passing across a smooth table at right angle to two opposite edges has two masses M_1 and M_2 ($M_1 > M_2$) attached to its ends hanging vertically as shown in Fig. 8.15. If a mass M be attached to the portion of the string which is on the table, find the acceleration of the system when left to itself.*



Sol. Refer to Fig. 8.15.

Let T_1 and T_2 be the tensions in the two portions of the strings.

Acceleration of the system, *a* = ?

We know that

$$W_1 = M_1 g, \; W_2 = M_2 g$$

$\therefore$ Equations of motion are :

$$M_1 g - T_1 = M_1 a \quad ...(i)$$

$$T_1 - T_2 = M \cdot a \quad ...(ii)$$

$$T_2 - M_2 g = M_2 \cdot a \quad ...(iii)$$

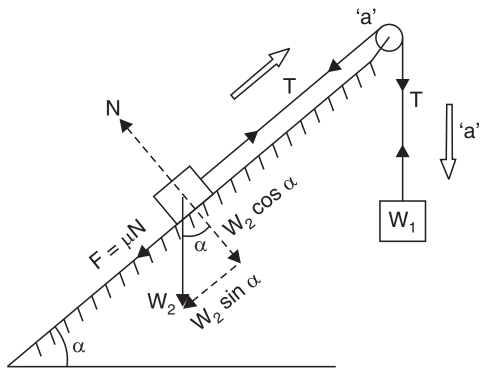
Adding (*i*), (*ii*) and (*iii*), we get

$$M_1 g - M_2 g = a\,(M_1 + M + M_2)$$

$$\boldsymbol{a} = \left[\frac{\mathbf{M_1 - M_2}}{\mathbf{M_1 + M + M_2}}\right] \times \mathbf{g}. \quad \textbf{(Ans.)}$$

. MOTION OF TWO BODIES CONNECTED BY A STRING ONE END OF WHICH ISHANGING FREE AND THE OTHER LYING ON A ROUGH INCLINED PLANE

Fig. 8.16 shows two bodies of weight W_1 and W_2 respectively connected by a light inextensible string. Let the body 1 of weight W_1 hang free and body 2 of weight W_2 be placed on an inclined rough surface. The velocity and acceleration of the body 1 will be the same as that of body 2. Since the string is inextensible, therefore, tension will be same throughout.



Let a = acceleration of the system

α = inclination of the plane

μ = co-efficient of friction between body and the inclined surface

T = tension in the string.

Consider the motion of body 1 :

Forces acting on it are : W_1 (downwards), T (upwards)

Resultant force = $W_1 - T$ ...(*i*)

Since the body is moving downwards, therefore force acting on the body

$$= \frac{W_1}{g} . a \quad ...(ii)$$

Equating (*i*) and (*ii*)

$$W_1 - T = \frac{W_1}{g} . a \quad ...(1)$$

Now consider the motion of body 2 :

Normal reaction at the surface,

$$N = W_2 \cos \alpha$$

∴ Force of friction, $F = \mu N = \mu W_2 \cos \alpha$

The forces acting on the body 2 as shown are :

T (upwards), $W \sin \alpha$ (downwards)

and $F = \mu W_2 \cos \alpha$ (downwards)

∴ Resultant force = $T - W_2 \sin \alpha - \mu W_2 \cos \alpha$ ...(*iii*)

Since, this body is moving along the inclined surface with acceleration therefore force acting on this body

$$= \frac{W_2}{g} a \quad ...(iv)$$

Equating (*iii*) and (*iv*), we get

$$T - W_2 \sin \alpha - \mu W_2 \cos \alpha = \frac{W_2}{g} a \quad ...(2)$$

Adding equations (1) and (2), we get

$$W_1 - W_2 \sin \alpha - \mu W_2 \cos \alpha = \frac{a}{g} (W_1 + W_2)$$

$$\therefore \qquad a = \frac{g\,(W_1 - W_2 \sin\alpha - \mu W_2 \cos\alpha)}{W_1 + W_2}$$

Substituting this value of 'a' in equation (1), we get

$$W_1 - T = \frac{W_1}{g}\,a$$

$$T = W_1 - \frac{W_1}{g}\,a = W_1\left(1 - \frac{a}{g}\right)$$

$$= W_1\left[1 - \frac{W_1 - W_2 \sin\alpha - \mu W_2 \cos\alpha}{W_1 + W_2}\right]$$

$$= W_1\left[\frac{W_1 + W_2 - W_1 + W_2 \sin\alpha + \mu W_2 \cos\alpha}{W_1 + W_2}\right]$$

$$= W_1 W_2\left[\frac{1 + \sin\alpha + \mu\cos\alpha}{W_1 + W_2}\right]$$

i.e.,

$$T = \frac{W_1 W_2\,(1 + \sin\alpha + \mu\cos\alpha)}{W_1 + W_2}$$

For *smooth inclined surface* ; putting $\mu = 0$ in equations (8.13) and (8.14).

$$a = \frac{g\,(W_1 - W_2 \sin\alpha)}{W_1 + W_2}$$

and

$$T = \frac{W_1 W_2\,(1 + \sin\alpha)}{W_1 + W_2}$$

Example 8.24. *A body weighing 8 N rests on a rough plane inclined at 15° to the horizontal. It is pulled up the plane, from rest, by means of a light flexible rope running parallel to the plane. The portion of the rope, beyond the pulley hangs vertically down and carries a weight of 60 N at the end. If the co-efficient of friction for the plane and the body is 0.22, find:*

(*i*) *The tension in the rope,*

(*ii*) *The acceleration in m/s², with which the body moves up the plane, and*

(*iii*) *The distance in metres moved by the body in 2 seconds, starting from rest.*

Sol. Refer to Fig.

Let T newton be the tension in the string and a m/s² the acceleration of the system.

Considering motion of 60 N weight (W_1) :

$$60 - T = \frac{60}{g}\,.\,a \qquad ...(i)$$

Considering motion of 8 N weight (W_2) :

$$T - W_2 \sin\alpha - F = \frac{W_2}{g}\,.\,a$$

$$T - 8\sin\alpha - \mu N = \frac{8}{g}\,.\,a$$

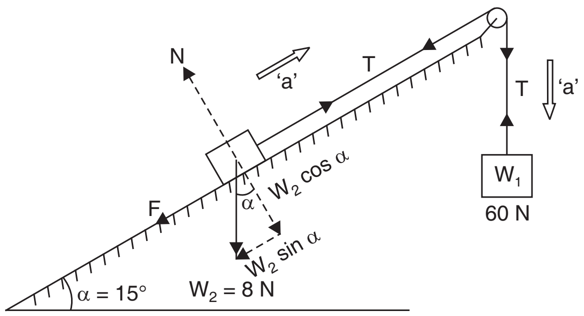


Fig. 8.17

$$T - 8 \sin \alpha - 0.22 \times 8 \cos \alpha = \frac{8}{g} \cdot a \qquad (\because \quad N = W_2 \cos \alpha = 8 \cos \alpha) \quad ...(ii)$$

Adding (*i*) and (*ii*)

$$60 - 8 \sin \alpha - 0.22 \times 8 \cos \alpha = \frac{68}{g} \cdot a$$

$$60 - 8 \sin 15° - 1.76 \cos 15° = \frac{68}{9.81} \times a$$

$$60 - 2.07 - 1.7 = \frac{68}{9.81} \times a$$

$\therefore$ $\quad$ $\boldsymbol{a = 8.11}$ **m/s²**. **(Ans.)**

Substituting this value of '*a*' in equation (*i*), we get

$$\boldsymbol{T} = 60 - \frac{60}{9.81} \times 8.11 = \mathbf{10.39\ N.\ (Ans.)}$$

Distance moved in 5 seconds, *s* = ?

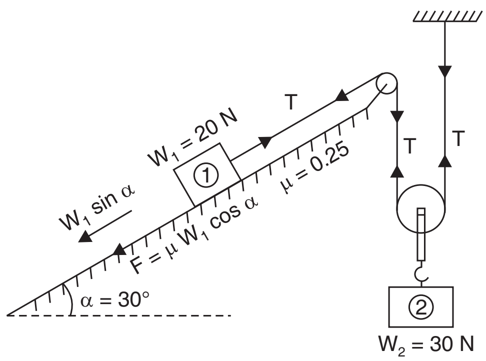
Initial velocity, $\quad u = 0$

Time, $\quad t = 2$ s.

Using the relation : $\quad s = ut + \frac{1}{2} at^2$

$\therefore$ $\quad \boldsymbol{s} = 0 + \frac{1}{2} \times 8.11 \times 2^2 = \mathbf{16.22\ m.\ (Ans.)}$

Example 8.25. *Determine the resulting motion of the body '1' assuming the pulleys to be smooth and weightless as shown in Fig. . If the system starts from rest, determine the velocity of the body '1' after 5 seconds.*



Sol. Weight of body '1', $W_1 = 20$ N

Weight of body '2', $\quad W_2 = 30$ N

Let $\quad T$ = tension in the string, and

$\quad a$ = acceleration of the body '1'.

Considering the motion of body '1' :

$$T - W_1 \sin \alpha - \mu W_1 \cos \alpha = \frac{W_1}{g} a$$

or $$T - 20 \sin 30° - 0.25 \times 20 \cos 30° = \frac{20}{g} \times a$$

or $$T - 10 - 4.33 = \frac{20}{g} a$$

or $$T - 14.33 = \frac{20}{g} a \qquad ...(i)$$

Considering the motion of body '2' :

A little consideration will show that the acceleration of body '2' will be half the acceleration of body '1' (*i.e., a*/2).

$$\therefore \qquad 30 - 2T = \frac{30}{g} \times \frac{a}{2} = \frac{15}{g} a \qquad ...(ii)$$

Multiplying eqn. (*i*) by 2 and adding eqns. (*i*) and (*ii*), we get

$$1.34 = \frac{55}{g} a$$

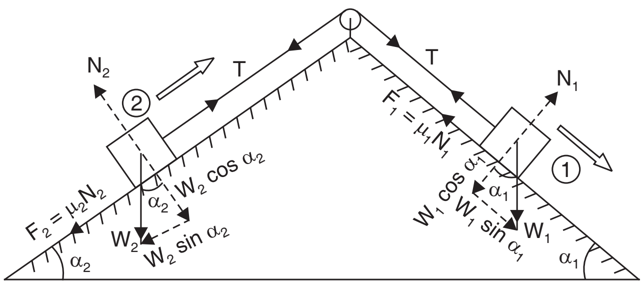
$$\therefore \qquad a = \frac{1.34 \times g}{55} = \frac{1.34 \times 9.81}{55} = 0.239 \text{ m/s}^2$$

∴ Velocity of body '1' after 5 sec., if the system starts from rest,

$\mathbf{v} = u + at = 0 + 0.239 \times 5 =$ **1.195 m/s. (Ans.)**

8.14. MOTION OF TWO BODIES CONNECTED OVER ROUGH INCLINED PLANES

Fig. shows two bodies of weight W_1 and W_2 respectively resting on the two inclined planes with inclinations α_1 and α_2 respectively.



Let a = acceleration of the system

μ_1 = co-efficient of friction between body 1 and the inclined plane 1 and

μ_2 = co-efficient of friction between body 2 and the inclined plane 2.

Consider the motion of body 1 :

Normal reaction at the surface,

$$N_1 = W_1 \cos \alpha_1$$

∴ Force of friction, $F_1 = \mu_1 N_1 = \mu_1 W_1 \cos \alpha_1$

The forces acting on body 1 are :

T (upwards), force of friction F_1 (upwards) and $W_1 \sin \alpha_1$ (downwards) as shown in Fig. 8.19.

∴ Resultant force $= W_1 \sin \alpha_1 - T - \mu_1 W_1 \cos \alpha_1$ ...(*i*)

Since this body is moving downwards, the force acting on this body

$$= \frac{W_1}{g} \cdot a \qquad ...(ii)$$

Equating (*i*) and (*ii*)

$$W_1 \sin \alpha_1 - T - \mu_1 W_1 \cos \alpha_1 = \frac{W_1}{g} \cdot a \qquad ...(1)$$

Now consider motion of body 2 :

Normal reaction at the surface,

$$N_2 = W_2 \cos \alpha_2$$

$\therefore$ Force of friction, $F_2 = \mu_2 N_2 = \mu_2 W_2 \cos \alpha_2$

The forces acting on body 2 are :

T (upwards), force of friction of F_2 (downwards) and $W_2 \sin \alpha_2$ (downwards) as shown in Fig.

Resultant force $= T - W_2 \sin \alpha_2 - \mu_2 W_2 \cos \alpha_2$ ...(*iii*)

Since the body is moving upwards, the force acting on the body

$$= \frac{W_2}{g} a \quad \text{...(iv)}$$

Equating (*iii*) and (*iv*)

$$T - W_2 \sin \alpha_2 - \mu_2 W_2 \cos \alpha_2 = \frac{W_2}{g} a \quad \text{...(2)}$$

Adding eqns. (1) and (2), we get

$$W_1 \sin \alpha_1 - W_2 \sin \alpha_2 - \mu_1 W_1 \cos \alpha_1 - \mu_2 W_2 \cos \alpha_2 = \frac{a}{g} (W_1 + W_2)$$

$$\therefore \quad a = \frac{g\,(W_1 \sin \alpha_1 - W_2 \sin \alpha_2 - \mu_1 W_1 \cos \alpha_1 - \mu_2 W_2 \cos \alpha_2)}{W_1 + W_2} \quad \text{...(8.17)}$$

Substituting this value of 'a' in equation (1), we get

$$W_1 \sin \alpha_1 - T - \mu_1 W_1 \cos \alpha_1$$

$$= \frac{W_1 \times g}{g} \times \frac{(W_1 \sin \alpha_1 - W_2 \sin \alpha_2 - \mu_1 W_1 \cos \alpha_1 - \mu_2 W_2 \cos \alpha_2)}{W_1 + W_2}$$

$$\therefore \quad T = (W_1 \sin \alpha_1 - \mu_1 W_1 \cos \alpha_1) - \frac{W_1 (W_1 \sin \alpha_1 - W_2 \sin \alpha_2 - \mu_1 W_1 \cos \alpha_1 - \mu_2 W_2 \cos \alpha_2)}{W_1 + W_2}$$

or

$$T = \frac{1}{(W_1 + W_2)} [(W_1 + W_2)(W_1 \sin \alpha_1 - \mu W_1 \cos \alpha_1) - W_1 (W_1 \sin \alpha_1 - W_2 \sin \alpha_2 - \mu_1 W_1 \cos \alpha_1 - \mu_2 W_2 \cos \alpha_2)]$$

$$= \frac{1}{(W_1 + W_2)} \times [W_1^2 \sin \alpha_1 - \mu_1 W_1^2 \cos \alpha_1 + W_1 W_2 \sin \alpha_1 - \mu_1 W_1 W_2 \cos \alpha_1 - W_1^2 \sin \alpha_1 + W_1 W_2 \sin \alpha_2 + \mu_1 W_1^2 \cos \alpha_1 + \mu_2 W_1 W_2 \cos \alpha_2]$$

$$= \frac{1}{W_1 + W_2} (W_1 W_2 \sin \alpha_1 + W_1 W_2 \sin \alpha_2 - \mu_1 W_1 W_2 \cos \alpha_1 + \mu_2 W_1 W_2 \cos \alpha_2)$$

$$= \left[\frac{W_1 W_2 (\sin \alpha_1 + \sin \alpha_2) - W_1 W_2 (\mu_1 \cos \alpha_1 - \mu_2 \cos \alpha_2)}{W_1 + W_2} \right]$$

$$= \frac{1}{W_1 + W_2} [W_1 W_2 (\sin \alpha_1 + \sin \alpha_2) - W_1 W_2 (\mu_1 \cos \alpha_1 - \mu_2 \cos \alpha_2)]$$

i.e.,

$$T = \frac{W_1 W_2}{W_1 + W_2} (\sin \alpha_1 + \sin \alpha_2 - \mu_1 \cos \alpha_1 + \mu_2 \cos \alpha_2) \quad ...(8.18)$$

For *smooth inclined plane* : putting $\mu_1 = 0$ and $\mu_2 = 0$ in equations (8.17) and (8.18), we get

$$a = \frac{g (W_1 \sin \alpha_1 - W_2 \sin \alpha_2)}{W_1 + W_2} \quad ...(8.19)$$

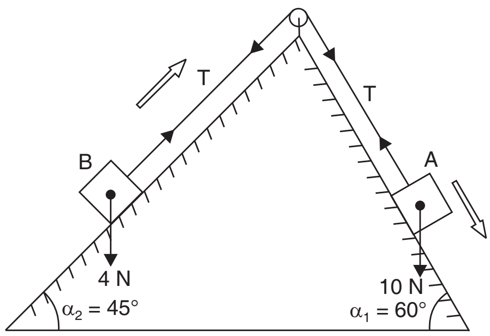
and

$$T = \frac{W_1 W_2}{W_1 + W_2} (\sin \alpha_1 + \sin \alpha_2) \quad ...(8.20)$$

26. *Blocks A and B weighing 10 N and 4 N respectively are connected by a weightless rope passing over a frictionless pulley and are placed on smooth inclined planes making 60° and 45° with the horizontal as shown in Fig. . Determine :*

(*i*) *The tension in the string and*

(*ii*) *Velocity of the system 3 seconds after starting from rest.*



Sol. Refer to Fig.

Let 'T' be the tension in the rope and 'a' the acceleration of the system.

(*i*) Tension, T = ?

For block A :

Resolving forces *parallel* to the plane :

$$10 \sin 60° - T = \frac{10}{g} . a \quad ...(i)$$

For block B :

Resolving forces parallel to the plane,

$$T - 4 \sin 45° = \frac{4}{g} . a \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$10 \sin 60° - 4 \sin 45° = \frac{14}{g} . a$$

$$8.66 - 2.83 = \frac{14}{9.81} \times a$$

$$a = 4.08 \text{ m/s}^2$$

Substituting this value of equations 'a' in (*i*), we get

$$10 \sin 60° - T = \frac{10}{9.81} \times 4.08$$

$$\therefore \quad T = 10 \sin 60° - \frac{10}{9.81} \times 4.08$$

$= 8.66 - 4.16 =$ **4.5 N. (Ans.)**

(*ii*) Velocity after 3 seconds, *v* = ?

Using the relation : $v = u + at$

$= 0 + 4.08 \times 3 \qquad (\because \; u = 0)$

= **12.24 m/s. (Ans.)**